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ABSTRACT

This paper was presented at the 1979 meeting of the American Educational Research Association (AERA) in San Francisco. It describes specific examples of calculator-aided mathematics learning activities for children in grades 2-6. These examples illustrate a variety of topics and types of learning which can be enhanced by a range of capabilities of a four-function calculator. Activity sheets and explanations of their uses are included. The activities are categorized by their instructional goals, and are designed to teach counting and numeration, basic facts, some selected number concepts, estimation, and problem solving. A discussion of the role of the calculator in mathematics learning as illustrated by the sample activities is also presented. (HM)

Calculators in Mathematics

How Should They Be Used

by

Harold L. Schoen

University of Iowa

A paper in the symposium entitled, "The Effects of Calculator Availability on School Mathematics Curriculum" at the 1979 Annual AERA Meeting, San Francisco, California.

The main purpose of this paper is to describe specific examples of calculator-aided mathematics learning activities appropriate for children in grades 2 through 6 which evolved from the Calculators in Elementary School Mathematics project. These examples have been chosen to illustrate a variety of topics and types of learning which can be enhanced by a range of capabilities of a four-function calculator. Research on the effects of using sequences of such activities with children and the implications for curricular change will be discussed by other participants in this symposium.

Activity sheets and explanations of their uses are included in the next sections. The activities are categorized by their instructional goals. The next five sections contain activities designed to teach counting and numeration, basic facts, some selected number concepts, estimation, and problem solving, respectively. The last section contains a discussion of the role of the calculator in mathematics learning as illustrated by the sample activities.

Counting and Numeration

By using the constant addend feature of most inexpensive calculators, children can count forward or backward by any specified number such as ones, twos, fives or tens. For young children (grades 2 or 3), simply counting to 1000 and watching the display show the number at each step is a good activity. Children get a feel for the size of 1000. For example, when asked about her progress when she reached 810, one child hesitated and responded, "You mean I just have to go through the 800's and 900's and then I'm there?" It was as if a light had flashed in her head.

Activity 1 illustrates counting sequences with various starting numbers and step sizes. The goal of this activity is to have students learn the types of number sequences which result from skip counting. The calculator provides visual verification of the result at each step. Numeration concepts are especially evident in sequences, where the step sizes are multiples of ten or one hundred. A good teacher would discuss the sequences in light of the base ten numeration system. Eventually children should be able to predict the next term in a sequence before pressing the = sign.

Activity 2 is a follow-up to Activity 1. On the surface, this appears to be just another pattern recognition activity. However, children may be able to complete a sequence like

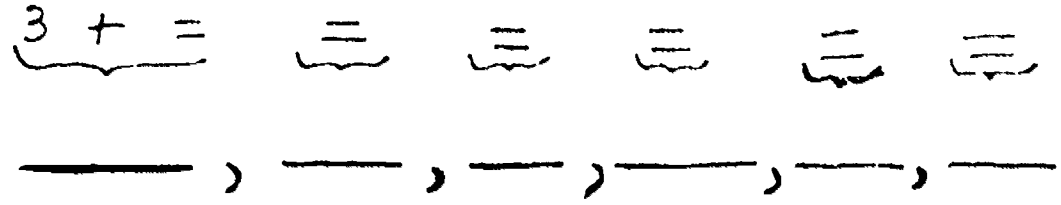
100, 200, 300, _____, _____, _____, _____

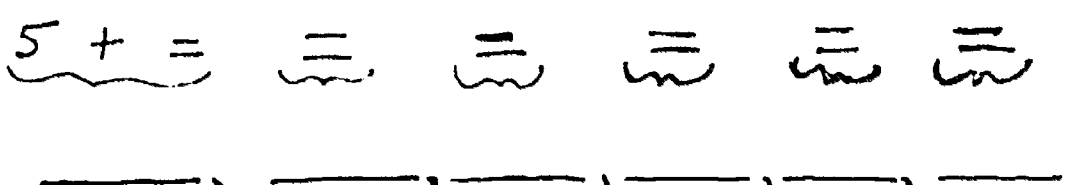
without knowing that it is generated by the formulas,

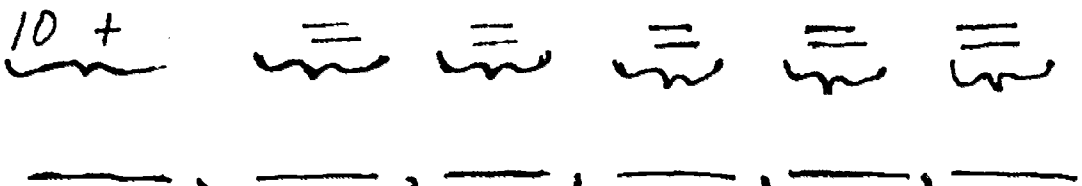
$$s_1 = 100; \quad s_i = s_{i-1} + 100 \text{ for } i > 1.$$

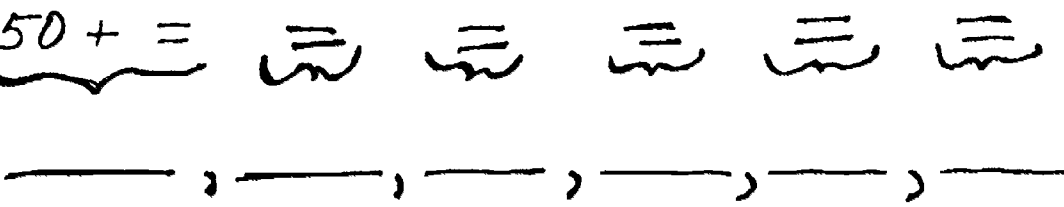
The calculator's contribution is to ensure that the student has not just recognized a perceptual pattern, but that s/he is able to generate the number sequence recursively using mathematical operations.

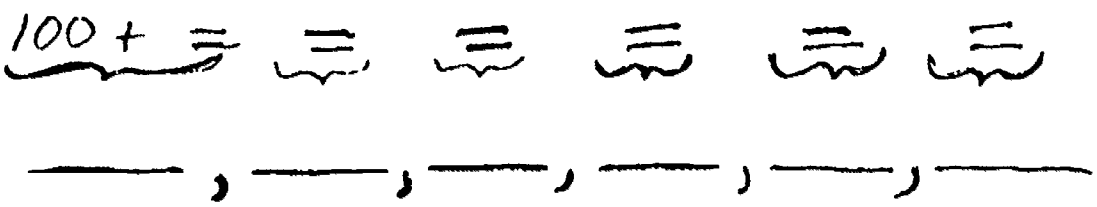
The objectives of Activities 3 and 4 are more directly related to the numeration system. Once children learn the sequence of basic digits, 0 through 9, they must also learn how to form larger numerals. A difficult aspect of this is the rule for writing numerals which follow those of the form 10k - 1 where k is any whole number. In Activity 3, children must predict the next numeral and then display it on the calculator. The power of the calculator is evident here as it provides a visual display of the result and enables children to experiment with a broad range of

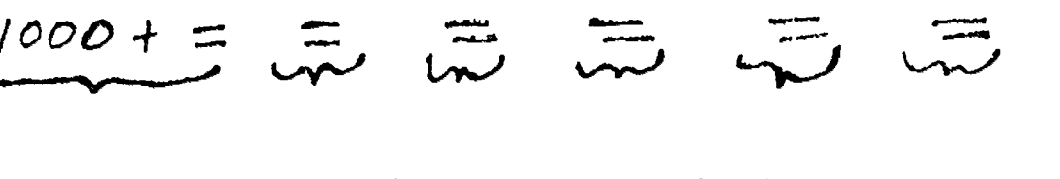
1. 3 + = 

2. 5 + = 

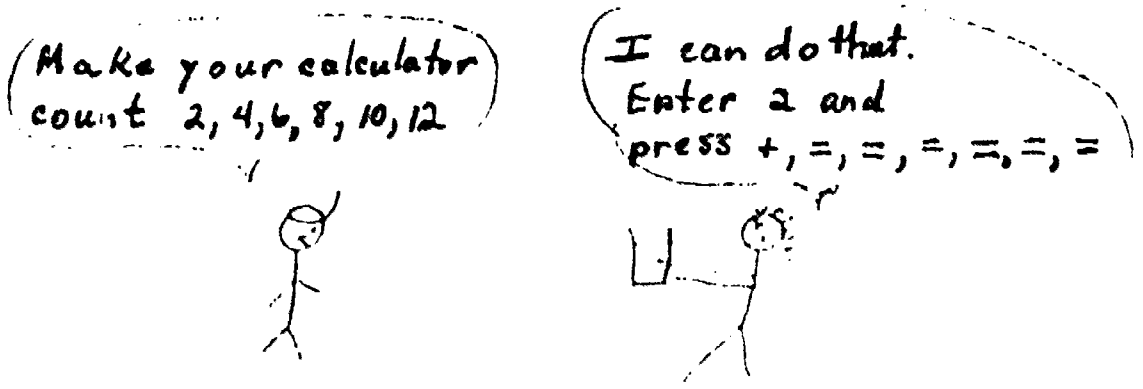
3. 10 + 

4. 50 + = 

5. 100 + = 

6. 1000 + = 

MAKE YOUR CALCULATOR COUNT



Make your calculator count like this. Fill the blanks.

1. 1, 2, 3, 4, 5, 6, 7, 8, ___, ___, ___, ___

2. 4, 8, 12, 16, 20, ___, ___, ___, ___, ___

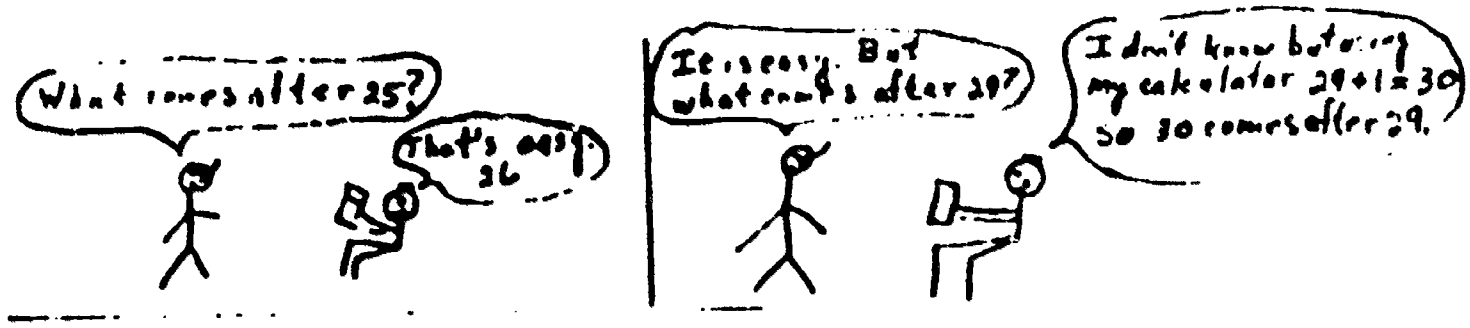
3. 5, 10, 15, 20, ___, ___, ___, ___, ___, ___

4. 20, 40, 60, 80, ___, ___, ___, ___

5. 40, 80, 120, 160, ___, ___, ___

6. 100, 200, 300, ___, ___, ___, ___

11. The Number After



Find the number after this one. Use your calculator to check.

		My Answer	Calculator Answer
1.	39	☐	39 + 1 = ☐
2.	59	☐	59 + 1 = ☐
3.	99	☐	99 + 1 = ☐
4.	109	☐	109 + 1 = ☐
5.	190	☐	190 + 1 = ☐
6.	199	☐	199 + 1 = ☐
7.	289	☐	289 + 1 = ☐
8.	299	☐	299 + 1 = ☐
9.	590	☐	590 + 1 = ☐
10.	999	☐	999 + 1 = ☐

instances of this generalization. Some obvious extensions of this activity are to find the "number before" e.g. 100 - 1 = 99, and to find the "number that is 10 more" e.g. 990 + 10 = 1000.

In Activity 4, children must make a zero appear in place of a specified digit in a multidigit whole number by subtracting a well-chosen number. This activity can be made into a two-player game in which players alternately challenge each other to change a digit to zero in a number. Even young children begin to see how to do this quickly. The activity can be made more complex by entering larger numbers and by requiring that more than one digit be changed to zero. The calculator provides the visual display, rapid computation and facility with large numbers to make this activity an effective way to teach about place value.

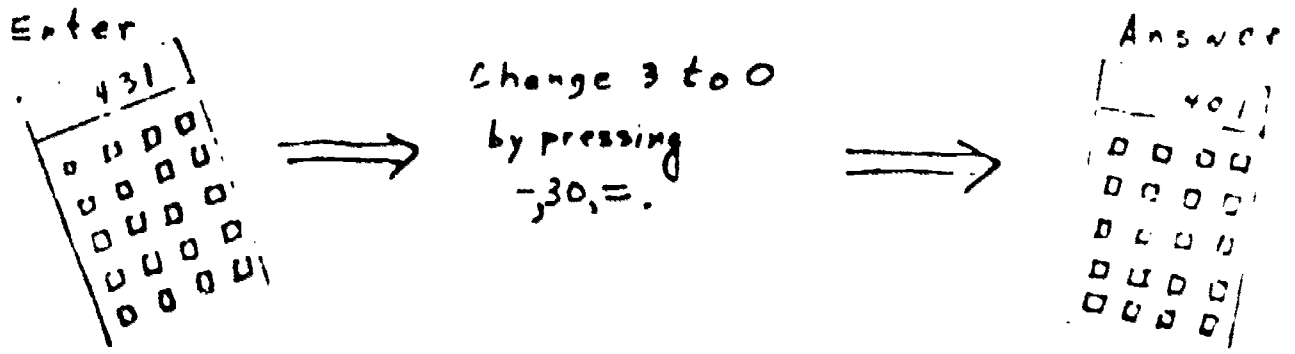
Basic Facts

The area of the mathematics curriculum which is of greatest concern to critics of calculator usage with young children is basic arithmetic facts. Activity 5 shows how the calculator can be used to teach basic addition facts. Preliminary research results indicate that this is a very efficient means of drilling the facts. The immediate visual feedback and the easily individualized practice are probably the main strengths of the activity. A simple adjustment of operations can make this a source of drill for multiplication, and subtraction facts as well.

Sample Concept Development

The calculator lends itself well to a counting-on model of addition and a counting-back model of subtraction. These models are illustrated in Activities 6 and 7, respectively. Many calculator-aided experiences

15. CHANGE A DIGIT TO ZERO



Try these.

	Enter	Make Zero	Keys Pressed	Answer
1.	672	7	___ , ___ , ___	___
2.	384	8	___ , ___ , ___	___
3.	9761	6	___ , ___ , ___	___
4.	9761	7	___ , ___ , ___	___
5.	9761	1	___ , ___ , ___	___
6.	2536	5	___ , ___ , ___	___
7.	97541	7	___ , ___ , ___	___

Basic Facts
Addition

Fact Check

Name _____

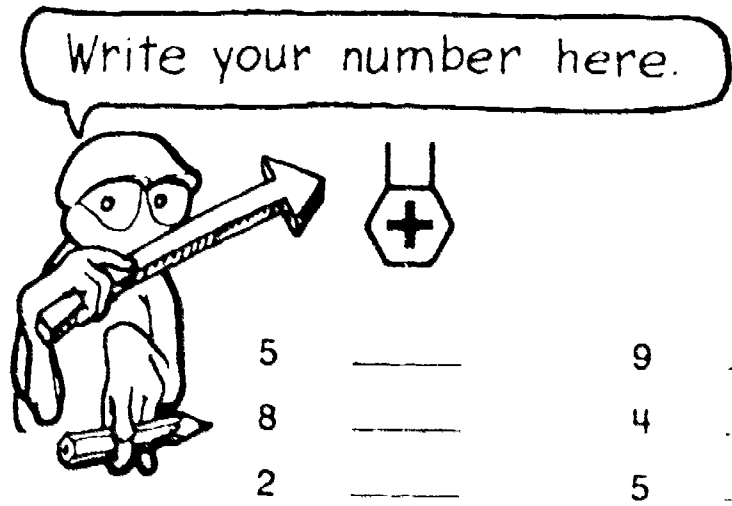
7 + 4 = 11
2 + 4 = 6
8 + 4 =

Here's a quick way to use your 🖩 to check your answers.

- Enter (4)(+)(=). The 4 is now locked in your 🖩. Any number you enter will be added to 4.
- Enter (7)
- Give your answer. Say, "Seven plus four equals eleven."
- Press (=). If your answer matches the answer on the 🖩, mark an O.K. beside 7.
- Enter (2)
- Give your answer. (2 + 4 = ?)
- Press (=) to check your answer.
- Mark an O.K. if your answer is right, and an X if it is wrong.

7 O.K.
2 O.K.
8 _____

Try these. Choose the number that you want to check and write it in the space provided.



5	_____	9	_____	2	_____	6	_____
8	_____	4	_____	4	_____	2	_____
2	_____	5	_____	1	_____	9	_____
4	_____	7	_____	0	_____	1	_____
6	_____	6	_____	5	_____	4	_____
9	_____	8	_____	3	_____	8	_____
0	_____	9	_____	2	_____	5	_____
7	_____	8	_____	4	_____	3	_____
1	_____	6	_____	0	_____	0	_____
3	_____	7	_____	3	_____	7	_____

of this type should improve the children's understanding of the meaning of addition and subtraction.

An extension of the counting back idea leads naturally to negative numbers as illustrated in Activity 8. At first children are surprised to see negative numbers as they count back. Many even expect that by counting back they will eventually get back to zero, but they soon discard that idea as the number in the display becomes larger (in absolute value). After several activities of this type, most second graders are comfortable with the appearance of negative numbers on their display. Some even develop a good insight into the less than zero idea.

Related number facts which illustrate the inverse relationship of addition and subtraction are a part of most mathematics curricula in the early grades. The calculator can be used to provide a much broader range of examples of this concept than is otherwise possible. See Activity 9.

Estimation

Once the solution of a mathematics problem has been correctly conceived, the calculator is a very accurate computational tool. The major source of error is miskeying, which usually leads to a result very different from the correct one. Thus, the ability to make a good preliminary estimate, a skill often overlooked in the mathematics curriculum, becomes exceptionally important. Fortunately the calculator itself can be used to help develop estimation skills, as Activities 10 and 11 illustrate.

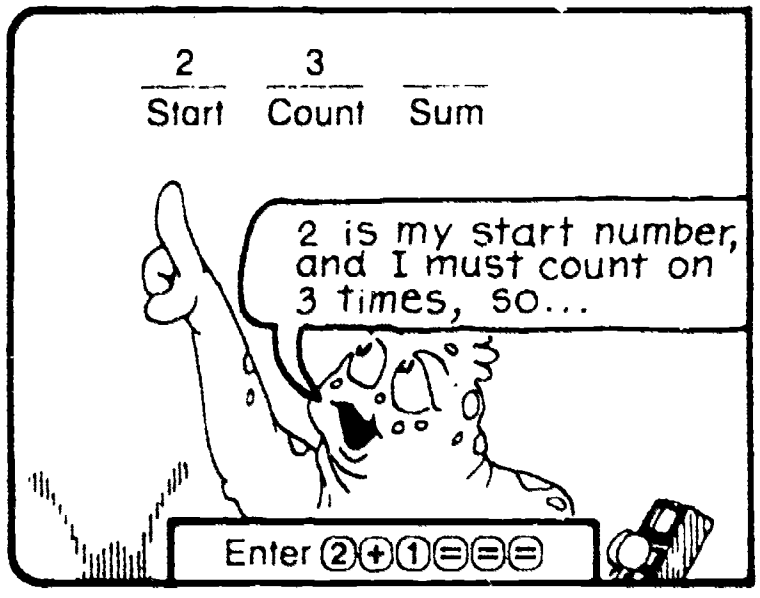
Activity 10 is analogous to golf, also a game of successively better estimates to the exact goal. This activity, as well as Activity 11, should be used following instruction on techniques of estimation and estimation activities with smaller numbers.

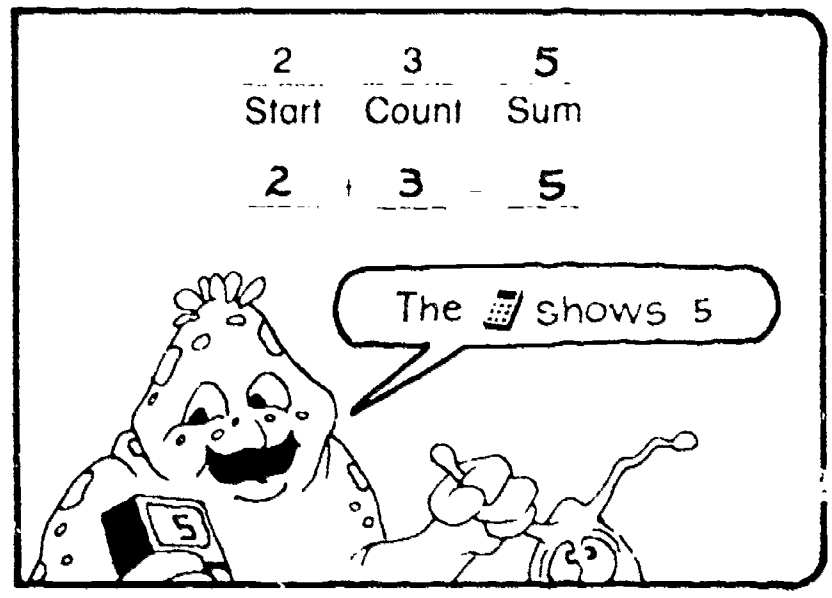
Counting On
Basic Facts

Addition Count

Name _____

Let's do some more counting!





Find each sum below. Then write the fact that goes with each sum.

1. 4 Start, 2 Count, ___ Sum

___ + ___ = ___

2. 5 Start, 3 Count, ___ Sum

___ + ___ = ___

3. 7 Start, 2 Count, ___ Sum

___ + ___ = ___

4. 8 Start, 3 Count, ___ Sum

___ + ___ = ___

5. 3 Start, 8 Count, ___ Sum

___ + ___ = ___

6. Start 6, Count 3, Sum ___ + ___

7. Start 7, Count 4, Sum ___ + ___

8. Start 6, Count 5, Sum ___ + ___

9. 18 Start, 2 Count, ___ Sum

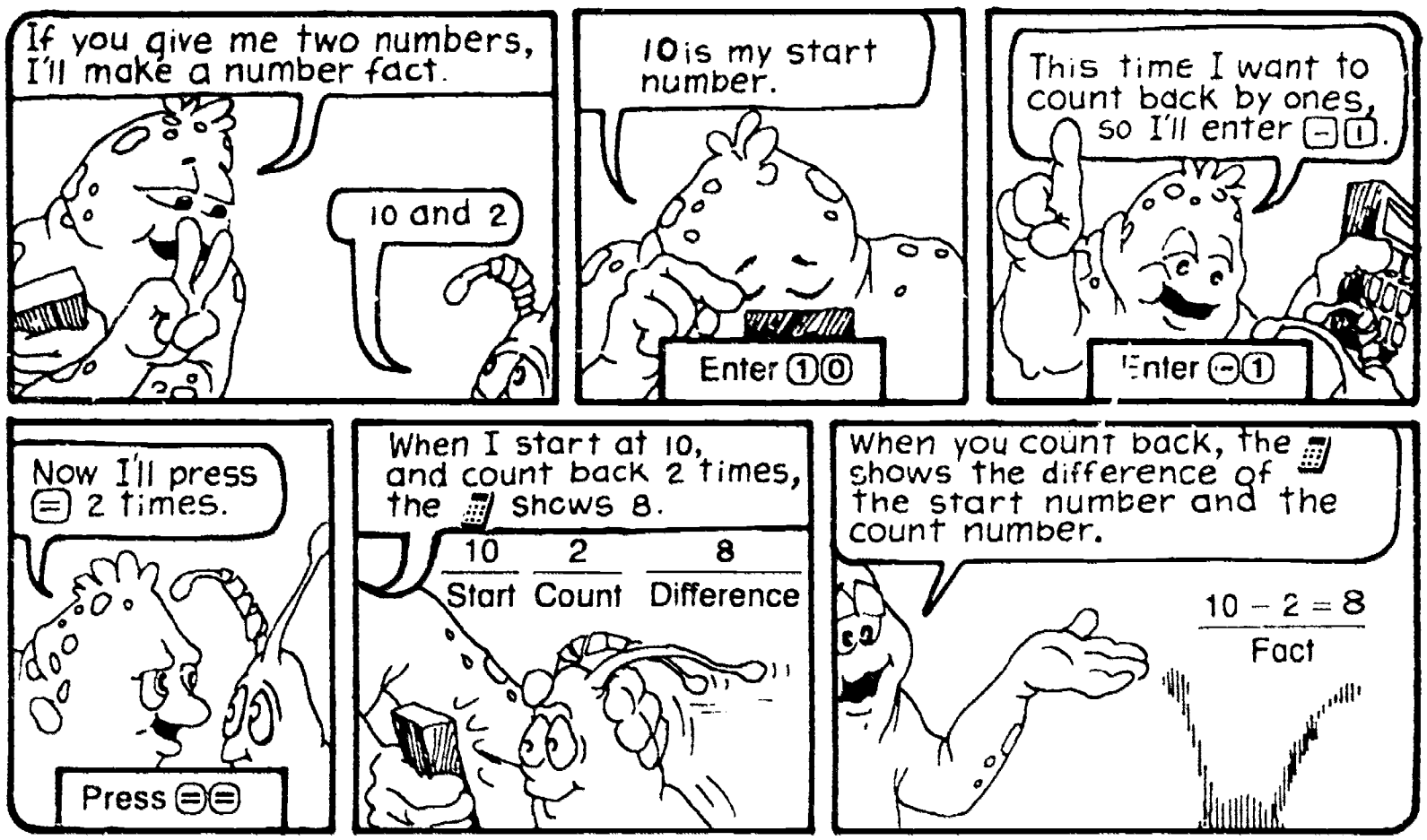
___ + ___ = ___

10. 98 Start, 2 Count, ___ Sum

___ + ___ = ___

Counting Back

Name _____



Try these.

1. 15 Start, 7 Count, 8 Difference
   15 − 7 = 8 Fact

2. 13 Start, 7 Count, _____ Difference
   13 − 7 = _____ Fact

3. 17 Start, 9 Count, _____ Difference
   17 − 9 = _____ Fact

4. 14 Start, 7 Count, _____ Difference
   14 − 7 = _____ Fact

5. 24 Start, 5 Count, _____ Difference
   24 − 5 = _____ Fact

6. 34 Start, 5 Count, _____ Difference
   34 − 5 = _____ Fact

7. 54 Start, 5 Count, _____ Difference
   54 − 5 = _____ Fact

8. 104 Start, 5 Count, _____ Difference
   104 − 5 = _____ Fact

Counting
Subtraction
Negative Numbers

Below Zero

Name _____

Fill in the blanks on the thermometer by counting back from 10. Enter ⑩⊖①⊜. Each time you press ⊜, write the display number on the thermometer.

Put your finger on the thermometer at 4. Count down 9 spaces. What number do you stop on? ____ On your 🖩, enter ④⊖⑨⊜. What number appears? ____

Use your 🖩 to solve each equation below. Then use your thermometer.

1. 9 − 5 = ____

Enter ⑨⊖⑤⊜

Start at 9.

Count down 5.

Stop at ____ .

2. 9 − 13 = ____ .

Start at 9.

Count down 13.

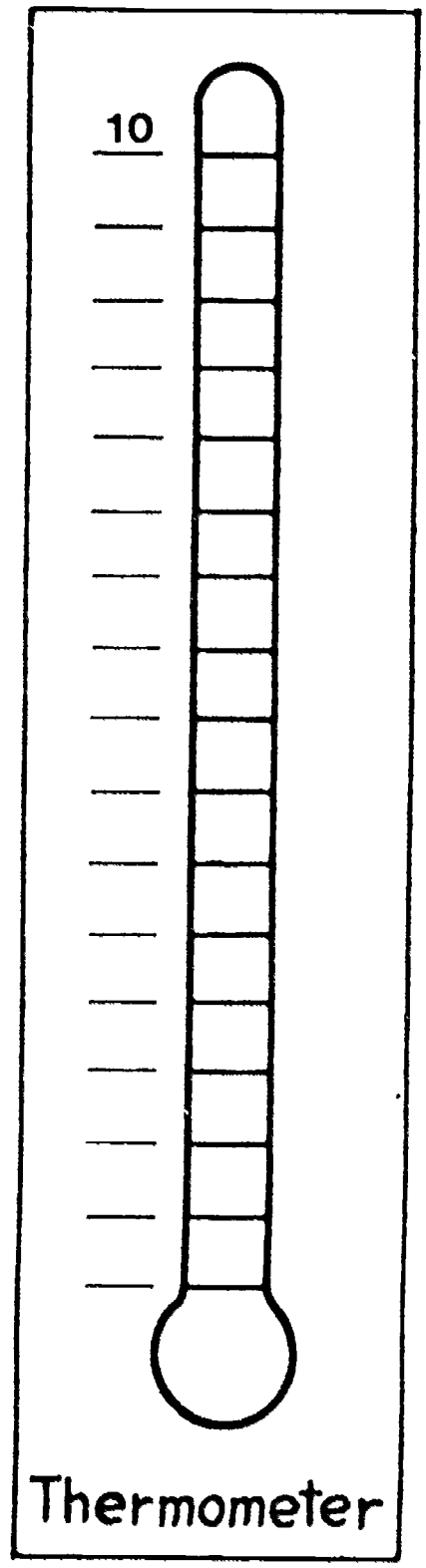
Stop at ____ .

3. 5 − 4 = ____

Start at 5.

Count down ____ .

Stop at ____ .



4. 4 − 5 = ____

Start at 4.

Count down ____ .

Stop at ____ .

5. 0 − 6 = ____

Start at ____ .

Count down ____ .

Stop at ____ .

6. 7 − 12 = ____ .

Start at ____ .

Count down ____ .

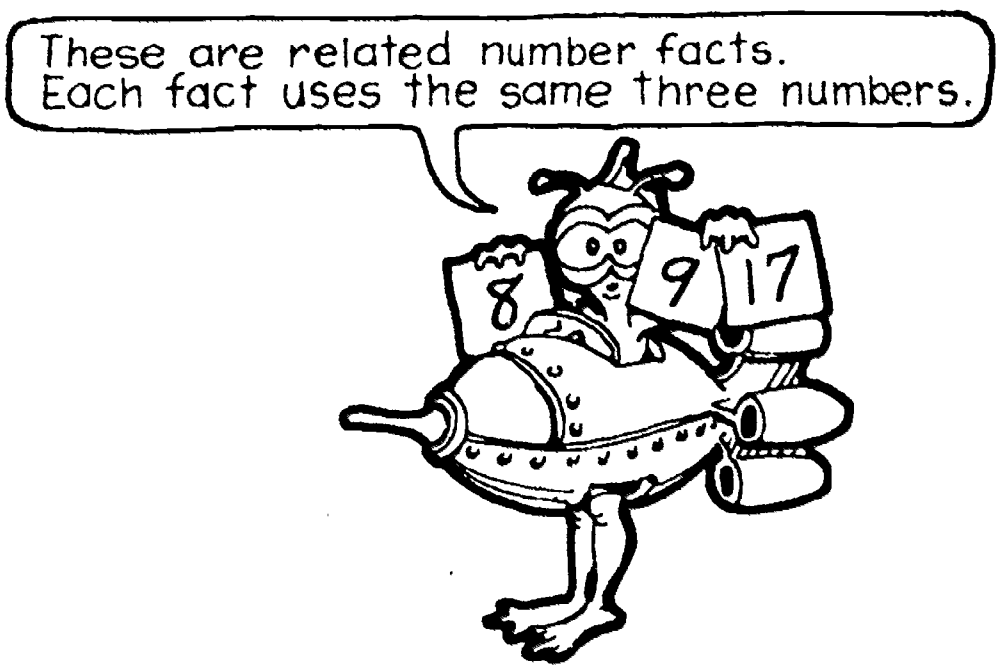
Stop at ____ .

Related Number Facts
Patterns

Fact Families

Name _____

Use your 🖩 to find the first fact in each row below. Then try to complete the remaining facts before checking on your 🖩.



9 + 8 = 17
8 + 9 = 17
17 − 8 = 9
17 − 9 = 8

1.	18 + 9 = ____	9 + 18 = ____	27 − 9 = ____	27 − 18 = ____
2.	35 + 18 = ____	53 − 18 = ____	18 + 35 = ____	53 − 35 = ____
3.	____ − 40 =	86 − ____ = 40	40 + ____ = 86	____ + 40 = 86
4.	100 − 63 = ____	100 − ____ = 63	63 + ____ = 100	37 + 63 = ____
5.	18 + ____ = 82	____ + 18 = 82	82 − 64 = ____	____ − 18 = 64
6.	99 + 49 = ____	148 − 49 = ____	49 + 99 = ____	148 − ____ = 49
7.	201 + 199 = ____	199 + ____ = 400	____ − 199 = 201	____ − 201 = 199

8. Make your own related number fact.

△ + □ = 60

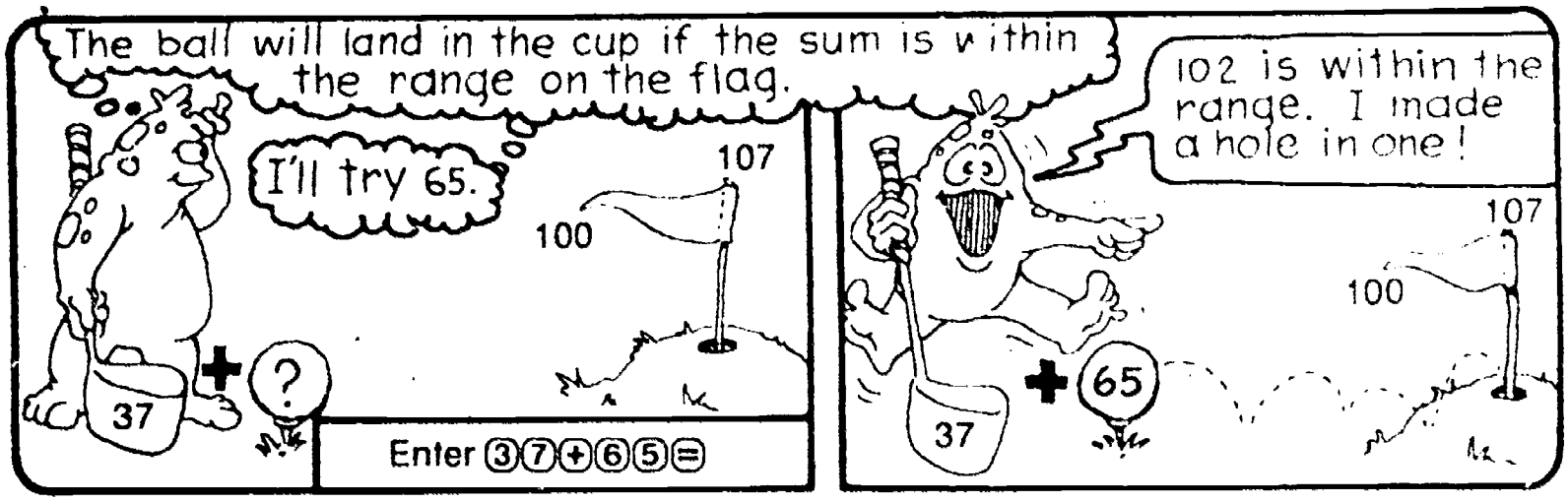
60 − □ = △

60 − △ = □

□ + △ = 60

Hole in One

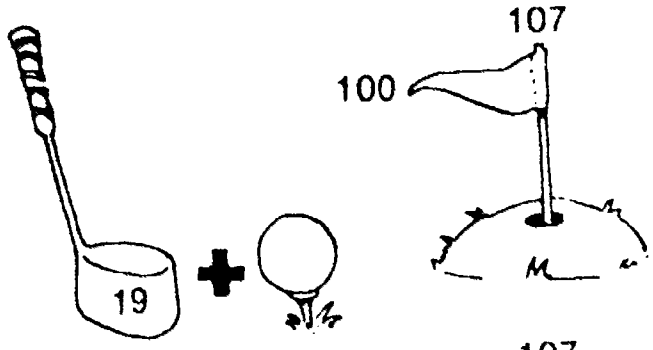
Name _____



Try these. Use your 🖩 to check if your sum is within the range. If your first estimate does not work, pick another number.

How many holes in one can you make?

1.



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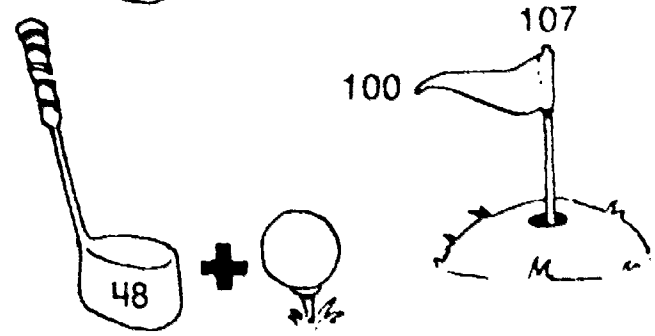

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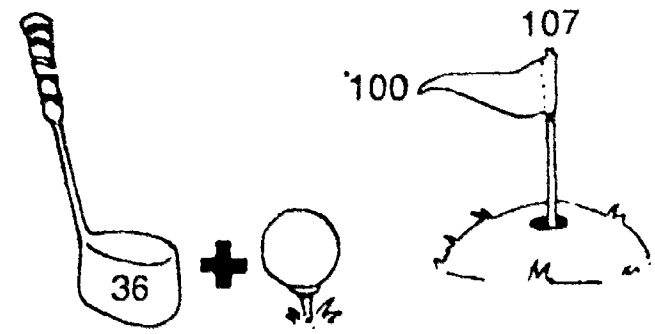

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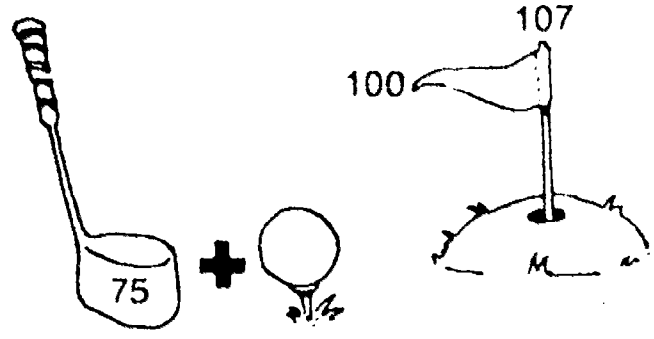

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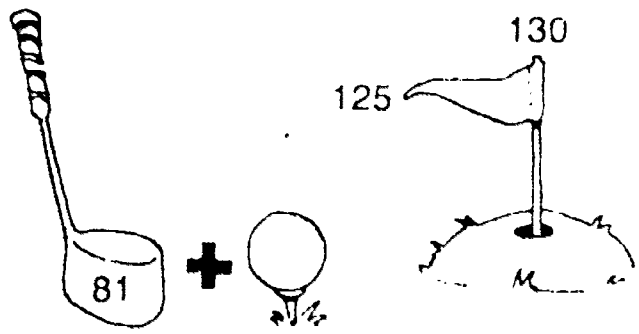

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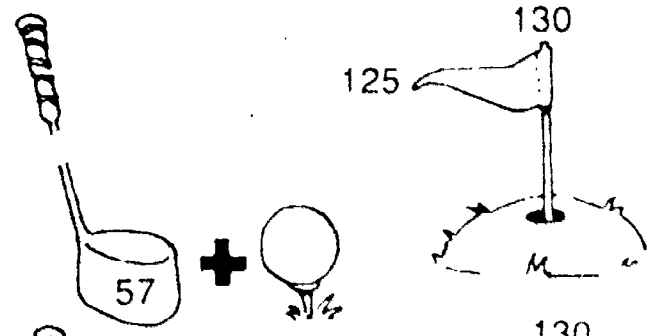

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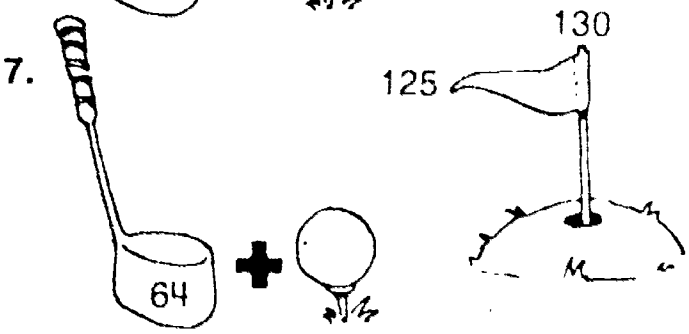

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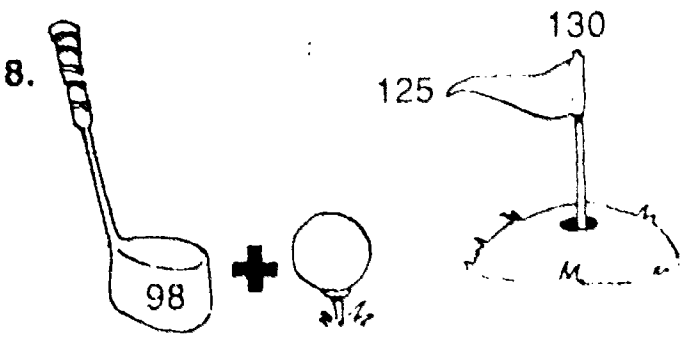


Activity 11 is a variation of Activity 10 which aims at developing skills at estimating in subtraction. The role of the calculator in both activities is to provide a means to easily verify estimates and quickly try new ones. To carry out the verification of successive estimates without a calculator would be so tedious as to make the activities useless.

Problem Solving

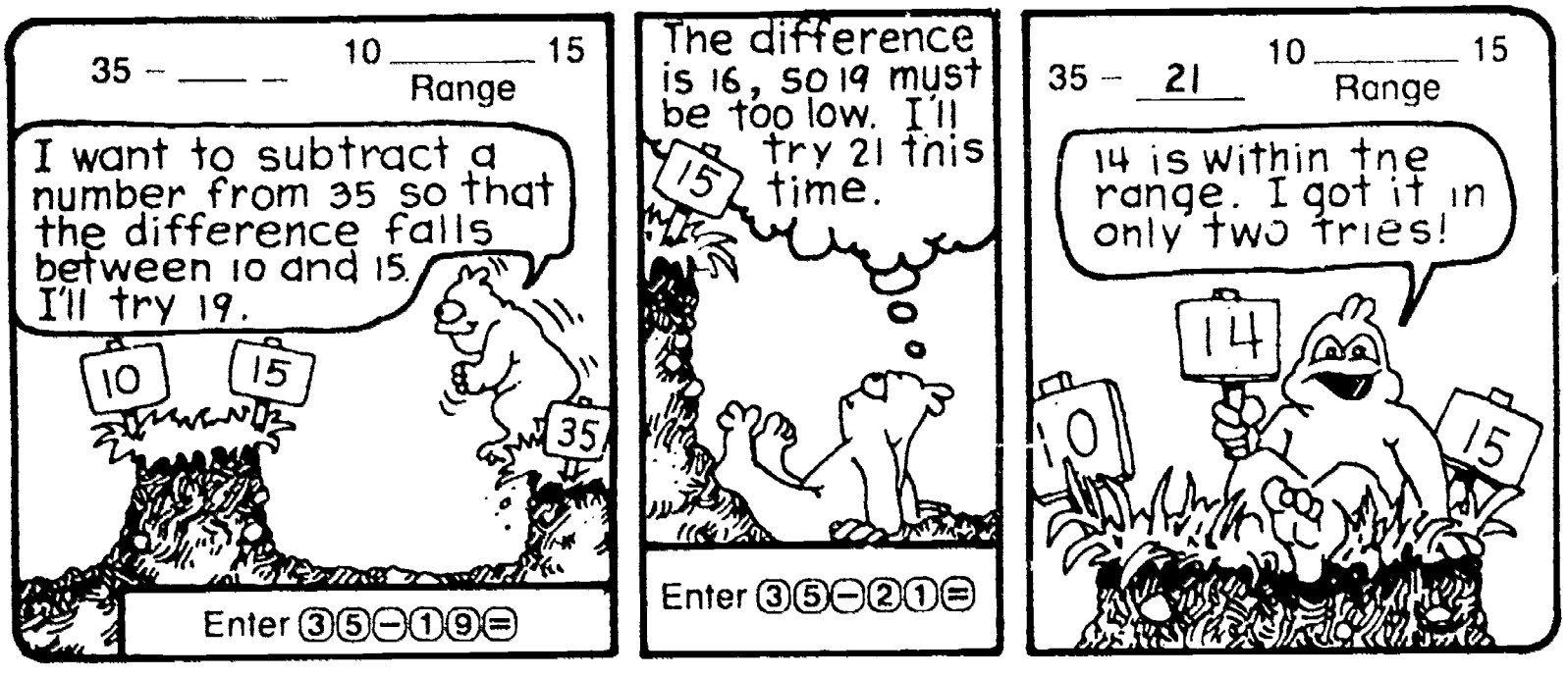
The calculator has tremendous, but for the most part unexplored, potential as a problem solving tool. The calculator makes it possible for students to use some heuristics which would be inaccessible or impractical without the calculator. For example, to determine how many buses seating 60 children each are needed to take all 406 children in a school on a field trip, several young children entered 406 in the calculator then proceeded to subtract 60 again and again. One child's explanation made it clear that she was imagining the buses filling one at a time, a very realistic conceptualization. Successive approximation is another heuristic that is useful but not practical if much paper and pencil computation is needed to check each approximation. A calculator eliminates that difficulty.

Another aspect of problem solving which the calculator can change is the need to use small numbers with the results "coming out even." Data which were the result of actual measurement, surveys, etc., were usually too messy to use in math. With a calculator, the need to avoid difficult computation is no longer a consideration. This enables teachers and curriculum developers to use interesting, real world problems with real data. Some examples of such problems are given in Activities 12 and 13.

Estimation
Subtraction

Range of Differences

Name _____



Try these. Use your 🖩 to check your guesses. Be sure to tally the number of tries each time.

1. 20 – ____ = ____ 8 ____ 12 Range

Number of tries: ____

2. 45 – ____ = ____ 5 ____ 10 Range

Number of tries: ____

3. 98 – ____ = ____ 40 ____ 45 Range

Number of tries: ____

4. 40 – ____ = ____ 10 ____ 15 Range

Number of tries: ____

5. 100 – ____ = ____ 20 ____ 25 Range

Number of tries: ____

6. 47 – ____ = ____ 15 ____ 19 Range

Number of tries: ____

7. 83 – ____ = ____ 50 ____ 55 Range

Number of tries: ____

8. 106 – ____ = ____ 10 ____ 15 Range

Number of tries: ____

9. 120 – ____ = ____ 95 ____ 99 Range

Number of tries: ____

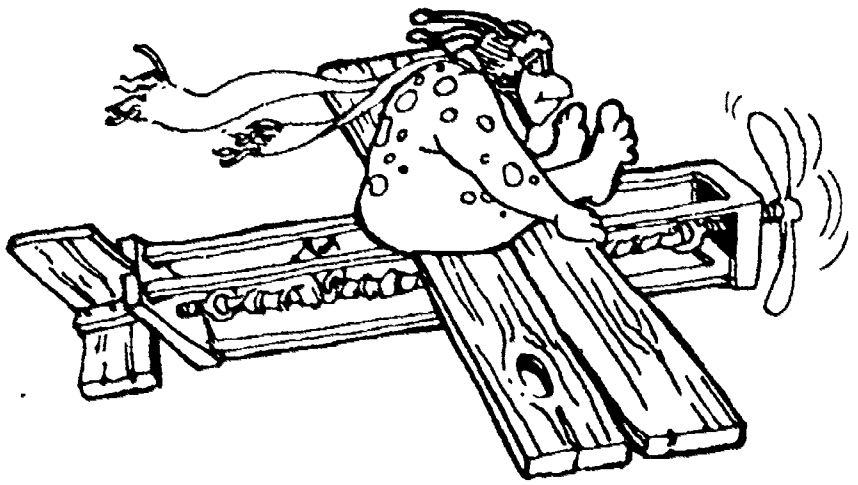
10. 110 – ____ = ____ 80 ____ 85 Range

Number of tries: ____

Addition and Subtraction
Multidigit Numbers

Around the World

Name _____



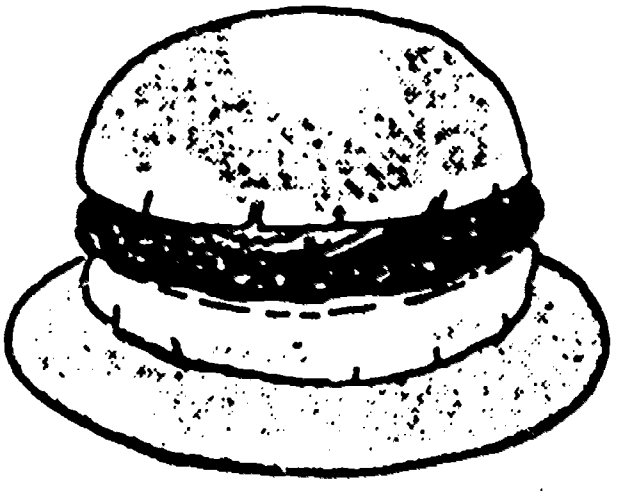
Date	From	To	Distance
Sept. 1	San Francisco	New York	4138 km
Sept. 2	New York	London	5582 km
Sept. 6	London	Bangkok	9546 km
Sept. 10	Bangkok	Tokyo	4610 km
Sept. 15	Tokyo	San Francisco	8286 km

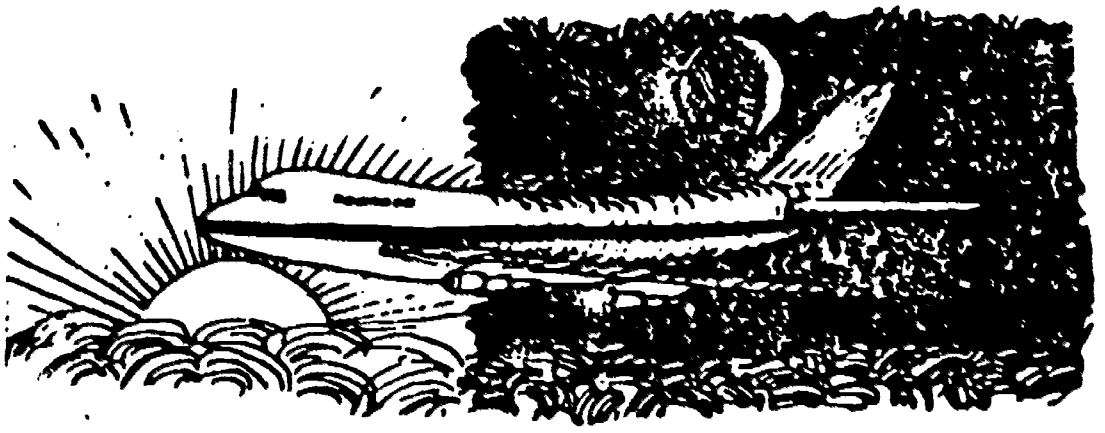
Use your 🖩 to answer the questions below.

1. What is the distance from San Francisco to London by way of New York?
   _____ km
2. How much farther is it from New York to London than from San Francisco to New York?
   _____ km
3. How much farther is it from London to Bangkok than from Bangkok to Tokyo?
   _____ km
4. What is the distance from San Francisco to Bangkok by way of New York and London?
   _____ km
5. What is the distance from San Francisco to Tokyo by way of New York, London, and Bangkok?
   _____ km
6. What is the total distance around the world if you travel from San Francisco to New York, London, Bangkok, Tokyo, and back to San Francisco?
   _____ km
7. How far is it from Tokyo to New York by way of San Francisco?
   _____ km
8. How far is it from Bangkok to San Francisco by way of Tokyo?
   _____ km

A HAMBURGER SOLD FOR 5 CENTS IN 1940. TODAY A SIMILAR HAMBURGER SELLS FOR 75 CENTS. IN 1940, A MAN EARNED 50 CENTS AN HOUR.

HOW MUCH SHOULD HE EARN TODAY IF SALARIES WENT UP AT THE SAME RATE AS HAMBURGERS?





A BOEING 747 SET A NEW RECORD FOR ITS CLASS OF JET OCTOBER 31, 1977, BY FLYING 26,706 MILES AROUND THE WORLD, OVER BOTH THE NORTH AND SOUTH POLES.

ON BOARD WERE 169 PASSENGERS. THOSE IN FIRST-CLASS SEATS HAD PAID $3,333 EACH. WHAT WAS THE COST PER MILE TO FLY FIRST-CLASS AROUND THE WORLD?

ACTIVITY 13

Summary and Discussion

Sample activities using the calculator as an aid to teach several elementary school topics were presented. The topics are counting and numeration, basic facts, estimation, problem solving and selected number concepts. These topics by no means exhaust those for which the calculator may be useful and obviously the included activities do not constitute complete sequences that would be necessary to teach the specified topics. The intent was to provide concrete examples of activities that would reflect a fairly broad range of topics which the calculator can be used to teach, and to show several different characteristics of the calculator that should be exploited for teaching mathematics.

In summary, the activities in this paper illustrate that the calculator can provide:

1. Rapid, accurate results of computation, even if large numbers and decimal fractions are involved.
2. Visual display of the results of mathematical operations.
3. A broad range of instances of mathematical concepts and generalizations.
4. A mechanical check of the student's understanding of mathematical symbolism and patterns.
5. A means for students to handle a broader range of numbers in problem solving settings, thereby enabling teachers to use more interesting and realistic problem settings.

Careful research on the effectiveness of instructional sequences utilizing these characteristics of the calculator is needed. Preliminary results from the Calculator in Elementary School Project, a broader,

feasibility study, suggest the cognitive and affective effects would be positive. The larger implications for elementary school mathematics curriculum reform must also be examined, debated and researched. One point is clear at this time--the calculator does provide several capabilities which can have a positive effect on curriculum and instruction in elementary school mathematics.

NOTES

1. Activities 5 through 12 are taken from:

   Reys, R. etal. Keystrokes Calculator Activities for Young Children. Addition and Subtraction. Palo Alto, California: Creative Publications, Inc., 1979.

2. Activity 13 is taken from unpublished materials prepared by the staff of the Iowa Problem Solving Project, directed by George Immerzeel.